GESTÃO, FINANÇAS E CONTABILIDADE

ANALYSIS OF REFERENCE OPTION PREMIUMS OF THE BRAZILIAN FUTURES EXCHANGE

UMA ANÁLISE DOS PRÊMIOS DE REFERÊNCIAS DA BOLSA DE FUTUROS BRASILEIRA

UN ANÁLISIS DE LOS PREMIOS DE REFERENCIA DE LA BOLSA DE FUTUROS DE BRASIL

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ABSTRACT
This paper compares the BM&FBovespa reference option premiums with the Garman-Kohlhagen model, Corrado-Su modified model, Merton's jump-diffusion model, and Black modified model for skewness and kurtosis for pricing dollar options and Ibovespa futures. Therefore, analysis scenarios were created and their results compared with the reference option premiums calculated by BM&FBovespa from January 2006 to November 2014. The results show that the reference option premiums calculated by BM&FBovespa are overvalued for dollar options. Regarding Ibovespa's future options, Merton's jump-diffusion model points to undervalued call premiums and overestimated put premiums. The discrepancy between the main estimation methods of the options and reference option premiums calculated by the stock exchange serves as a warning to investors who use reference option premiums in calculating their performance, due to the difficulty of measuring these values.
Keywords: Dollar Options, Ibovespa Futures Options, Black Model, Corrado-Su Modified Model, Merton Jump-Diffusion Model

RESUMO
O objetivo deste trabalho é comparar os prêmios de referência da BM&FBovespa e os modelos de Garman Kohlhagen, Corrado-Su Modificado, Difusão com Saltos de Merton e o modelo de Black adaptado para assimetria e curtose para a precificação de opções de dólar e de futuro de Ibovespa. Para isso, foram definidos cenários de análise e comparados os resultados com os prêmios de referência calculados pela BM&FBovespa no período janeiro de 2006 a novembro de 2014. Os resultados obtidos mostram que os prêmios de referência calculados pela Bolsa estão superestimados para as opções de dólar. Para as opções de futuro de Ibovespa, o modelo de Difusão com saltos de Merton indica uma subestimação do prêmio das opções de compra e uma superestimação do prêmio das opções de venda. A divergência entre os principais métodos de estimação do prêmio da opção e o prêmio de referência calculado pela bolsa de valores serve de alerta aos investidores que utilizam o prêmio de referência no cálculo do seu desempenho, devido a dificuldade em mensuração desses valores.
Palavras-chave: Opção de Câmbio, Opções de Futuro de Ibovespa, Modelo de Black, Modelo Corrado-Su Modificado, Modelos de Difusão com Saltos de Merton.

RESUMEN
El objetivo de este estudio es comparar los premios de referencia de la BM&FBovespa y modelos de Garman Kohlhagen, Corrado-Su Modificado, de difusión con saltos de Merton y el modelo de Black adaptado a la asimetría y curtosis para la fijación de precios de opciones de dólar y futuros de Ibovespa. Para este análisis se definieron escenarios y se compararon los resultados con las primas de referencia calculados por la Bolsa en el período comprendido

entre enero de 2006 y noviembre de 2014. Los resultados muestran que las primas de referencia calculados por la Bolsa están sobreestimadas para las opciones de dólar. Para las opciones de futuro de Ibovespa, el modelo de difusión con saltos Merton indica una subestimación de la concesión de opciones sobre acciones y una sobreestimación de la prima de las opciones de venta. La divergencia entre los principales métodos de estimación de la prima de la opción y la prima de referencia calculada por la bolsa de valores sirve como una advertencia a los inversores que utilizan la prima de referencia en el cálculo de sus rendimiento, debido a la dificultad de medir estos valores.
Palabras clave: Opción de Cambio, Futuros Opciones de Ibovespa, Modelo de Black, Modelo de Corrado-Su modificado, modelos de difusión con saltos de Merton.

1. INTRODUCTION

The role played by derivatives in the architecture and health of company financial management is notorious, as they can be negotiated in organized exchanges or on the so-called over-the-counter market. In the former, contracts follow standards, which facilitates negotiation between agents, such as in futures and option contracts. In the latter, however, there is a great deal of flexibility in negotiating contract features and conditions, such as in forward contracts, swaps, and flexible options (FARHI, 1999).

Technically, option contracts are agreements in which one party acquires the right to buy or sell an asset at a pre-set price up to a certain date and the counterparty is obliged to honor the buying or selling price for the asset in exchange for a sole initial payment, called the option premium (HULL, 2006).

The advantage of using exchange options instead of forward contracts is the greater flexibility of the former, since it guarantees to the corporation that the value of the currency will not be above (call option) or below (put option) the strike value for the option, whereas it allows the corporation to be the beneficiary of favorable shifts in exchange rates.

Forward contracts, on the other hand, fix the exchange rate to be used in the future transaction. The inherent disadvantage of using exchange options is the cost tied to the transaction, since, although forward contracts have negligible costs, exchange options are acquired by paying a premium (HULL, 2006).

Economic volatility introduces profound changes in agent behavior, clarifying the need for a solid understanding of expectations regarding the growth of main economic and financial variables, both in the short and long term. This trend toward hedging against market changes increased after the 2008 financial crisis, due to a greater demand for hedging derivatives instead of speculating purposes, as mentioned by Lopes, Schiozer, and Sheng (2013), and Coutinho, Sheng, and Lora (2012).

In Brazil, negotiating foreign exchange options contracts in US dollars began in 1988, and gradually increased in volume until it reached USD 800 million in 1998. In 1999, the growth rate stepped up with the adoption of the floating exchange rate system, which resulted in increased demand for protection against exchange rate changes (CUNHA JR.; LEMGRUBER, 2003). Moreover, foreign exchange is a major asset in the Brazilian economy and is inversely proportional to the stock market (BERNADELLI; BERNADELLI; CASTRO, 2017). Therefore, it is used to hedge against potential internal problems.

Several studies suggested different pricing models for exchange options, since traditional approaches are usually criticized for adopting empirically unverifiable assumptions. Moreover, there is evidence that there is a significant error in the pricing of this type of asset in the industry (BHARGAVA; BROOKS; MALHOTRA, 2001).

This pricing error is a major concern for individuals involved in derivatives market transactions, since, given the large volumes negotiated, even small differences can cause serious losses (BHARGAVA; BROOKS; MALHOTRA, 2001). Regarding the Brazilian market, although this type of contracts has existed for over two decades, very few studies on the effectiveness of the options evaluation models adopted by the market for pricing exchange options were carried out (CUNHA JR.; LEMGRUBER, 2003).

There are also futures options, where negotiations deal with the right to buy or sell a futures contract and not the asset itself. Consequently, the exercise of this option grants it a position in the futures market. The indirect processing for enacting the contract creates a false impression of inefficiency, when in fact, it enables greater liquidity in its strike.

Options on Ibovespa futures are negotiated at BM&FBovespa, which grants investors the right to take a position in the futures market of the Brazilian exchange index. As such, it enables investors to take advantage of an advanced trend, carry out defensive operations, or change how aggressive their asset portfolios are, with the flexibility of choosing whether to exercise their position on the option.

These contracts, both for foreign exchange options and options on Ibovespa futures, are negotiated within BM&FBovespa and, therefore, the exchange provides each with a reference premium. This premium is used as an index on balance sheets, since the value of the financial position a company may henceforth have on the market is quantified based on the reference premiums published. Consequently, it is desirable to monitor the impact that these positions might have on a company's health, so auditing firms pay special attention to these contracts.

Reference option premiums are also important inputs for investment funds, which use them in mark-to-market practice. The funds invested by the investor vary according to the share of the fund, which is defined by this premium. Prices of the securities that make up the fund's portfolio are stipulated in the index, based on the market value of these assets, which is defined by the value of the share.

Negotiation carries on to the options market regarding the exchange rate, which was introduced in 2011 with the aim of improving the Global Trading System (GTS) for this market. This mechanism enables the recording of bids and the closing of deals in real time. The GTS has a rejection tunnel for this purpose, which is aimed at reducing the occurrence of errors created by the rejection of erroneous bids placed outside the range of prices set by the exchange.

Thus, it is clear that it is of utmost importance that the reference option premiums provided by the exchange be correctly calculated, consistent with market variables and features.

The objective of this study is to carry out a comparison between the BM&FBovespa reference option premiums for foreign exchange and Ibovespa futures options with the Garman-Kohlhagen, Corrado-Su modified, Merton's jump-diffusion, and the skewness and kurtosis adjusted Black-Scholes models.

The models are suitable for quantifying the premiums on these contracts, and can be used as standards for comparison with the models used by the exchange. Therefore, the premiums calculated by the proposed models were compared to the reference option premiums calculated by BM&FBovespa from January 2006 to November 2014.

The comparison showed a deviation in the reference option premiums calculated by the exchange. They are overvalued for foreign exchange options, undervalued for Ibovespa futures call options, and overvalued for Ibovespa futures put options.

2. THEORETICAL REFERENCE

When proposing a pricing formula for European options on stocks that do not pay dividends, Black and Scholes roused market interest. However, major limitations of the model were gradually identified, which were based on the fact its hypotheses were frequently violated when real market data was analyzed. Ever since the Black-Scholes model was developed in 1973, several other researchers have put forward modifications and adjustments to better suit the formula to the market. Therefore, the model served as a basis for the development of several new models, which sought to sidestep its limitations. For instance, Garman and Kohlhagen (1983) extended the Black-Scholes model to currency options. Black (1976) developed the analytical formulas that enable the evaluation of futures contracts. In 1996, Corrado and Su incorporated skewness and kurtosis in the options pricing formula, and Merton (1976) considered the possibility of jumps in series of prices.

2.1. Options Pricing

The literature on options pricing dates to the beginning of the twentieth century, with studies by the French mathematician Louis Bachelier, who deduced an option pricing formula based on the assumption that stock prices follow a Brownian motion with zero drift. Since then, numerous researchers have contributed to the theory of option pricing (MERTON, 1973).

In the 1970s, Fischer Black and Myron Scholes (BLACK; SCHOLES, 1973) put forth their option pricing model. The model is based on a selection of hypotheses for pricing European stock options that do not pay dividends, where the premium of call or put options is a function of the price of the asset, strike price, life of the option, risk-free interest rate, and volatility of the asset price.

Although it represents a breakthrough in the theory of finance, the Black-Scholes model had limitations, since its hypotheses were violated upon analyzing market data. With the aim of improving the traditional Black-Scholes model, several authors have proposed expansions to approximate the distribution of the probability density of the implied asset return distribution. In this approach, skewness and kurtosis of the series may have significant impact on option prices, leading to the understanding that adjustments to the Black-Scholes model could result in the elimination of the biases observed.

The original Black-Scholes model assumes that the logarithm of asset prices follows a process of diffusion with constant variance. Previous studies carried out by Black and Scholes (1972) and Officer (1973) test and reject the validity of the premise of constant variance. Since then, the literature on heteroscedasticity that started with Engle’s (1982) and Bollerslev’s (1986) studies, documented the volatile nature of the variance of stock returns.

Now, it is known that the variance of stock returns is stochastic and correlated with the price of the asset, suggesting a non-normal distribution of returns. Consequently, Heston (1993), Hull and White (1987), Stein and Stein (1991), and Wiggins (1987) show that, under these conditions, the original Black-Scholes model is expected to systematically result in options premiums different from those observed.

Consequently, Corrado and Su (1996) propose incorporating skewness and kurtosis into the Black-Scholes model, an idea also reviewed by Jurczenko, Maillet, and Negrea (2004), who introduced a change in the original formula, thus modifying it for consistency.

In relation to the pricing of foreign exchange options, two models stand out: the Garman and Kohlhagen (1983) model and that of Black (1976). The former demonstrates that the options pricing model with stocks that constantly pay dividends could be used for pricing exchange rate derivatives, since the compensation from a position in foreign currency is similar to the payment of continuous dividends. The latter model can be used for pricing when the futures and the options expire on the same date (BLACK, 1976; MERTON, 1976). This model was originally proposed for incorporating the analytical formulas that enabled the evaluation of options on futures contracts.

The models with jumps relax the diffusion hypothesis in asset prices, thereby inputting jump content on the path followed by the asset. One of the main models in this class is that of Merton (1976), who assumes the existence of a discontinuity in the distribution of asset returns, which is modeled by a Poisson procedure.

On the other hand, few studies about foreign exchange options pricing have been conducted in the Brazilian market. Cunha Jr. and Lemgruber (2003) test the interest rate model and the stochastic foreign exchange coupon in evaluating foreign exchange options in the Brazilian market. Through a sample of options negotiated at the exchange from 1998 to 2001, Cunha Jr. and Lemgruber (2003) conclude that the model's adherence to market prices was good, even in situations where the volatility of the interest rates and foreign exchange coupons are high.

Costa and Yoshino (2004) conclude that Heston's model proved itself to be acceptable for the Brazilian foreign currency market, as it presented stable parameters in the periods with the least volatility in the foreign exchange market and some instability during periods of greater volatility. They introduced Heston's model by using closing rates from exchange options between 2002 and 2003 as data, and they calibrated the model through a matrix of implied volatilities for a group of banks that operate in the domestic market.

2.2. The Garman-Kohlhagen Model

The Black-Scholes model prices European call and put options that do not pay dividends. Its formula assumes that the asset remains in geometric Brownian motion with constant volatility. Equations 1 and 2 estimate the option call and put price, respectively:

$$c = S_0N(d_1) - Xe^{-rT}N(d_2), \quad (1)$$

$$p = Xe^{-rT}N(-d_2) - S_0N(-d_1), \quad (2)$$

$$d_1 = \frac{\ln {S_0}/{X} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{T}, \quad (4)$$

where:

- c is the call option premium;
- p is the put option premium;
- S_0 is the asset price on day zero;
- X is the asset strike price;

- T is the time until the option expires;
- σ is the volatility of the asset;
- r is the risk-free interest rate with continuous compounding projected until the option expiration;
- $N(d_1)$ is the accumulated probability of normal distribution until d_1;
- $N(d_2)$ is the accumulated probability of normal distribution until d_2.

The Garman-Kohlhagen model uses the equations above, but variable r is calculated by subtracting two rates, that is, $r = b = r_L - r_f$, where r_L is the local (Brazilian) interest rate in a continuous compounding system and projected until the option expiration, and r_f is the foreign interest rate, which is also under a continuous funded system and projected until the option's expiration (EKVALL; JENNERGREN; NÄSLUND, 1997).

2.3. Corrado-Su Modified Model

The original Corrado-Su model, proposed in 1996, incorporated a skewness and kurtosis item into the Black-Scholes model, which refers to the series of returns for the assets studied. The original equation had a typo that could be significant and was later corrected (BROWN; ROBINSON, 2002).

However, the Corrado-Su modified model was adopted in research (JURCZENKO; MAILLET; NEGREA, 2004), and adapted to provide consistency with the Martingale limitation, thereby avoiding distortion in the premiums calculated in case the returns presented a skewed and leptokurtic distribution, according to Jurczenko, Maillet and Negrea (2004).

Although the differences between the results from the models proposed in Jurczenko, Maillet, and Negrea (2004) are small in most cases, it can be economically significant for specific cases such as options with far off expiration dates or that are deep out-of-the-money, mainly when markets are turbulent.

The equations that estimate the option price are as follows:

$$c = S_0 N(d_1) - Xe^{-rT} N(d_2) + \mu_3 Q_3 + (\mu_4 - 3) Q_4, \quad (5)$$

$$p = c - S_0 e^{(b-r_L)T} + Xe^{-rT}, \quad (6)$$

$$Q_3 = \frac{1}{6(1+w)} S_0 \sigma \sqrt{T} \left(2\sigma\sqrt{T} - d\right) N(d), \quad (7)$$

$$Q_4 = \frac{1}{(24(1+w))} S_0 \sigma \sqrt{T} \left(d^2 - 3d\sigma\sqrt{T} + 3\sigma^2 T - 1\right) N(d), \quad (8)$$

$$w = \frac{\mu_3}{6} \sigma^3 T^{1,5} + \frac{\mu_4}{24} \sigma^4 T^2, \quad (9)$$

$$d = \frac{\left[\log\left(\frac{S_0}{X}\right) + \left[b + \frac{\sigma^2}{2}\right] T - \log(1+w)\right]}{\sigma\sqrt{T}}, \quad (10)$$

where:

- μ_3 is the skewness coefficient;
- μ_4 is the kurtosis coefficient;
- b is the interest differential between the domestic and foreign rates, just like in the Garman-Kohlhagen model.

2.4. Merton's Jump-Diffusion Model

One of the conditions that must be met for the classical Black-Scholes model to be valid is the dynamic of the asset return to follow a continuous path. This assumption does not always represent reality, such as in the return on a commodity. The behavior thus presents jumps over a continuous process. Merton's jump-diffusion model puts forth a formula based on the Black-Scholes model, which considers the most generic cases (MERTON, 1976).

Therefore, admitting the existence of discontinuities in the distribution of returns on an asset, the existence of two different components affecting the asset price is noticed. The continuous part is modeled by a standard Wiener process, whereas the discreet element (jumps) is modeled by a Poisson process, with a rate that reflects the number of jumps per unit of time (MERTON, 1976).

Merton's model, despite being more complex, presents a level of accuracy that is slightly superior to that obtained by the Black-Scholes model, as pointed out by the study carried out by Canabarro (1988) for three stocks on the Brazilian market.

The equations that estimate the option price are as follows:

$$\sum_{i=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^i}{i!}\left[S_0 N(d_1) - X e^{-rT} N(d_2)\right], \tag{11}$$

$$d_1 = \frac{\ln {}^{S_0}/_X + \left(r + \frac{1}{2}\sigma_i^2\right)T}{\sigma\sqrt{T}}, \tag{12}$$

$$d_2 = d_1 - \sigma_i\sqrt{T}, \tag{13}$$

$$\sigma_i = \left(z^2 + \delta^2\left(\frac{i}{T}\right)\right)^{0,5}, \tag{14}$$

$$z = (\sigma^2 - \lambda * \delta^2)^{0,5}, \tag{15}$$

$$\delta = \left(\frac{\gamma * v^2}{\lambda}\right)^{0,5}, \tag{16}$$

where:

- v is the total volatility, including jumps;
- λ is the expected number of jumps per year;
- γ is the percentage of the total volatility of the series explained by the jumps.

2.5. Modified Black Model

As previously mentioned, Black and Scholes (1973) proposed a model based on a set of hypotheses for calculating the European call and put options premium on stocks that did not pay dividends. Merton (1973) extended the model to include payment of stock dividends and Black (1976) developed the analytical formulas that enable assessment of the options on futures contracts. Here, instead of working with the price *spot*, S, the future price F is used:

$$c = [\, F\, N(d_1) - X\, N(d_2)]e^{-rT}, \tag{17}$$

$$p = [\, X\, N(d_2) - \, F\, N(d_1)]e^{-rT}, \tag{18}$$

where

$$\mathrm{d1} = \frac{\left[\ln\left(\frac{F}{X}\right)+\sigma^2\frac{T}{2}\right]}{\sigma\sqrt{T}}, \tag{19}$$

$$\mathrm{d2} = \mathrm{d1} - \sigma\sqrt{\mathrm{T}}. \tag{20}$$

When dealing with options on futures contracts, skewness and kurtosis are incorporated into the previously cited model through equations (21) and (22):

$$c = [\, F\, N(d_1) - X\, N(d_2)]e^{-rT} + \mu_3 Q_3 + (\mu_4 - 3)Q_4, \tag{21}$$

$$p = [\, X\, N(d_2) - \, F\, N(d_1)]e^{-rT} + \mu_3 Q_3 + (\mu_4 - 3)Q_4, \tag{22}$$

$$Q_3 = \frac{1}{6(1+w)} F\sigma\sqrt{T}\big(2\sigma\sqrt{T} - d\big)N(d), \tag{23}$$

$$Q_4 = \frac{1}{(24\,(1+w))} F\sigma\sqrt{T}\big(d^2 - 3d\sigma\sqrt{T} + 3\sigma^2 T - 1\big)N(d), \tag{24}$$

$$w = \frac{\mu_3}{6}\sigma^3 T^{1,5} + \frac{\mu_4}{24}\sigma^4 T^2, \tag{25}$$

$$\mathrm{d} = \frac{\left[\log\left(\frac{F}{X}\right)+\left[b+\frac{\sigma^2}{2}\right]T - \log(1+w)\right]}{\sigma\sqrt{T}}, \tag{26}$$

where b is the differential of interest between domestic and foreign rates, μ_3 the skewness coefficient, and μ_4 the kurtosis coefficient.

3. METHOD

The goal of this study is to compare the BM&FBovespa premium option index with the foreign exchange and Ibovespa futures options available for every day analyzed, with data obtained from the Garman-Kohlhagen, Corrado-Su modified, Merton jump-diffusion, and Black adjusted models. In the last model, the adjustment is responsible for incorporating skewness and kurtosis into the model.

The study covers the period ranging from 01/01/2006, the first day that the premium was published, until 11/03/2014. For this purpose, on each first workday of all analyzed months, nine scenarios were created for analysis, as shown in Table 1:

Table 1- Examples of the scenarios created for each reference date

	In-the-money	At-the-money	Out-of-the-money
Expiration in 1 month	Scenario 1	Scenario 2	Scenario 3
Expiration in 2 months	Scenario 4	Scenario 5	Scenario 6
Expiration in 3 months	Scenario 7	Scenario 8	Scenario 9

The classification of in-, at-, and out-of-the-money is attributed to options according to the relationship between strike (X) and cash price (S) at the time of finalizing the contract. In the call options, if X is less than S, the option is called in-the-money, if X is equal to S, at-the-money, and if X is greater than S, out-of-the-money (FIGUEIREDO, 2010). Regarding the put options, the greater than and less than relationships between X and S are reversed.

Subsequently, data collection was carried out. For the exchange options, the base historic series was number 10813—US dollar call, u.m.c./USD—from the daily temporary series belonging to the Central Bank, dated 12/30/2005 to 11/28/2014, daily, obtained through the website of the institution.

Regarding the Ibovespa futures options, the base historic series used was number 7—Bovespa, point—from the temporary series of the Central bank for the same period. Once the data had been obtained, a Napierian logarithm was calculated for the return on the asset.

Additionally, a list of call and put options with different strike prices and expiration dates were obtained on the BM&FBovespa website for the first workday of the months ranging from January 2006 to November 2014. Up to nine options were chosen for each date so that the sample could include one in-the-money, one at-the-money, and another out-of-the-money option with expiration dates in 30, 60, and 90 days.

In some instances, it was impossible to obtain nine options, since those found in the report did not meet the assumptions for fitting into one of the scenarios or due to the inexistence of negotiations. For each of the options chosen, the following data were collected: lifetime left until expiration, strike price, and the reference option premiums calculated by BM&FBovespa.

Regarding the pricing for Ibovespa futures options, the Future price of the contracts was also collected. To avoid data loss, even if no negotiations took place, which would not affect the quality of the research, we used adjusted stock market quotes. This is a reliable number, since it represents the Ibovespa demand, in addition to the projected rate increase for interbank deposit (ID) futures until the expiration of the contract, as to avoid arbitration.

In total, for the dollar options, 823 call option and 782 put option contracts were assessed. Regarding Ibovespa futures options, 305 call option and 251 put option contracts were evaluated.

American (clean price) and Brazilian (interbank deposits x fixed rate) reference rates, both of which are in effect for 252 workdays of the year for 1, 2, and 3 months, were obtained on a Bloomberg terminal using the codes BCSWAPD, BCSWBPD, BCSWCPD CURNCY, and LIMCU30, LIMCU60, and LIMCU90 INDEX, respectively.

The foreign exchange coupon is calculated based on the difference between the internal interest rate and the exchange rate variation, thus being an interest rate in dollars. The foreign exchange coupon used as a reference on the day before the expiration for calculating foreign exchange variation is called the dirty price. The foreign exchange coupon without the differential is called the clean price, thus avoiding the distortion created by the differential between the rate and the adjustment (BOLSA DE MERCADORIA E FUTUROS, BM&F, 2007). Based on the data surveyed, the parameters to be applied to the analyzed models were then calculated. The calculations were obtained through programming in Microsoft Excel's Visual Basic Application (VBA).

Configuring the Garman-Kohlhagen model began by calculating the volatility on the returns of the respective assets (dollar and Ibovespa) with the following equation:

$$\sigma \text{ annual} = \sigma \text{ daily} * \sqrt{252}, \tag{27}$$

where daily volatility was calculated as a standard deviation of the series of returns, with data between the first and last workday of the month before the reference month. For example, for the month of September 2010 (09/01/2010), the data series considered pertained to the data ranging from 08/02/2010 to 08/31/2010.

The time until expiration was calculated by considering the number of workdays in the interval between the first workday of the reference month and the workday before the expiration date of the scenario being analyzed. The calculation is based on the workday before the expiration dates, since the closing price of that date could define the option's strike price.

The American and Brazilian continuous capitalization interest rates used in the model were calculated, respectively, as follows:

$$r_f = \ln(1 + \text{clean price}), \tag{28}$$

$$r_L = \ln(1 + \text{ID fixed rate}). \tag{29}$$

In the Corrado-Su modified model, in addition to the parameters determined in the Garman-Kohlhagen model, skewness and kurtosis were also calculated. Skewness was calculated by considering the data from the series of returns between the first and last workday of the month before the reference month. Likewise, kurtosis was calculated by considering the same data series used in calculating skewness.

In Merton's jump-diffusion model, besides the parameters determined in the Garman-Kohlhagen model, it was necessary to calculate the expected number of jumps per year, volatility without jumps, and the percentage of volatility of the series explained jumps. Total volatility considering the jumps, a parameter necessary for the model, is the same value used in the Garman-Kohlhagen and Corrado-Su modified models.

The expected number of jumps per year was calculated by considering two criteria:

- Values outside the range within two standard deviations;
- Values outside the range within three standard deviations.

Calculation of the averages and standard deviations were based on the complete series considered, and a quantity of jumps were identified for each criterion and applied to all scenarios, regardless of the reference date. These two criteria led to two analyses that consider the diffusion model with jumps.

The interval studied consist of 107 months. As such, the number of jumps found in each of the criteria had to be adjusted by a factor (12/107) to be able to represent the expected number of jumps per year.

Table 2 – Number of jumps found in the dollar series according to each criterion studied

Average	4.046E-05
Standard deviation	0.00953

Jumps:	
> 2 SD	58
< -2 SD	49
> 3 SD	22
< -3 SD	12
Jumps:	
First criterion	12.00
Second criterion	3.81

Table 3 – Number of jumps found in the Ibovespa series according to each criterion studied

Average	2.232E-4
Standard deviation	0.01838
Jumps:	
> 2 SD	49
< -2 SD	51
> 3 SD	14
< -3 SD	17
Jumps:	
First criterion	11.21
Second criterion	3.47

Calculating the volatility without jumps (σ') is necessary so that, later, the volatility of the series explained by the jumps can be calculated. This variable was obtained by excluding the jumps from the series of returns and recalculating the monthly volatility with the previously-used equation. Once again, one value is obtained per reference month.

The percentage of volatility explained by the jumps for each of the criteria is given by equation (30):

$$Percentage\ \sigma\ \text{explained by jumps} = 1 - \frac{\sigma\ \text{annual}'}{\sigma\ \text{annual}}. \tag{30}$$

This variable points out how much of the volatility of the series studied can be attributed to the observed jumps and, consequently, the impact of the jumps on the series can be determined.

Finally, in Black's adapted model, the parameters surveyed are used in the Garman-Kohlhagen model and skewness and kurtosis are calculated for the Corrado-Su modified model.

4. RESULTS

4.1. Dollar Options

The first comparison drawn was between the BM&FBovespa reference option premium and the premium calculated by the proposed models, seeking to verify whether the reference option premium was above or below that obtained from the models. Tables 4 and 5 show the average and standard deviations of this comparison. The results found for the dollar options are as follows:

Table 4 – Results of the comparison of values calculated by the models with the BM&FBovespa reference option premiums for dollar call options

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	97.63%	94.85%	97.51%	97.48%
Std. deviation	15.19%	16.51%	15.09%	15.07%
At-the-money	86.33%	83.92%	85.90%	85.83%
Std. deviation	22.17%	22.04%	22.08%	22.06%
Out-of-the-money	64.77%	63.47%	64.32%	64.24%
Std. deviation	34.23%	33.19%	33.91%	33.87%
Total average	81.25%	79.16%	80.90%	80.84%
Std. deviation	29.27%	28.69%	29.14%	29.12%

Table 5 – Results from the comparison of the values calculated by the models with the BM&FBovespa reference option premiums for dollar put options

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	89.52%	88.71%	89.23%	89.18%
Std. deviation	22.53%	22.39%	22.40%	22.38%
At-the-money	87.60%	84.54%	86.88%	86.77%
Std. deviation	40.56%	41.30%	40.26%	40.21%
Out-of-the-money	83.93%	36.24%	83.95%	83.90%
Std. deviation	61.37%	251.61%	60.63%	60.48%
Total average	86.98%	69.84%	86.64%	86.57%
Std. deviation	44.66%	149.13%	44.20%	44.11%

Note that, on average, the three scenarios (in-, at-, and out-of-the money) show lower results than those calculated by the BM&FBovespa reference option premiums.

For call options, the relative differences are more pointed for the out-of-the-money dollar options, where the average percentages are around 64%, i.e., a premium that is 36% lower than the index, whereas smaller relative differences are observed for the in-the-money dollar options, which present premiums consistent with those of the BM&FBovespa.

For put options, a greater average deviation is also noted for the out-of-money dollar options, where there are premiums around 84% of the reference premium, except for the value calculated with the Corrado-Su modified model, which is lower than the others.

Analyzing the scenarios, although in the call options the difference between the models and the reference option premiums shows a growth trend the more out-of-the-money the contract is, in the put options, the scenarios do not have a significant effect. The standard deviation trends, however, are clear for both call and put options. They point to greater variability in the calculated premiums the more out-of-the-money the contract is.

Attention must also be drawn to the fact that, when comparing the results among the models, the average premiums of the three models are similar for each scenario, except for the out-of-the-money put option calculated by the Corrado-Su modified model.

In this scenario, the ratio does not follow the pattern of the other models. The explanation for this is related to the methodology of the model, in which, according to Jurczenko, Maillet, and Negrea (2004), this model can reveal significant anomalies when the options are very out-of-the-money and in turbulent markets.

Another way of analyzing the results is to compute the values while bearing in mind the expiration of each option, thus shedding light on a possible effect of the expiration date on the difference between the reference option premiums and options premium models. The results of this perspective are shown in Tables 6 and 7.

Table 6 – Comparing values calculated by the models with the BM&FBovespa reference option premiums for the dollar call options considering the options' expiration dates

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
Expiration		In-the-money		
1 month	96.13%	93.67%	95.98%	95.95%
2 months	97.31%	94.11%	97.19%	97.17%
3 months	100.63%	97.82%	100.54%	100.52%
Expiration		At-the-money		
1 month	84.92%	82.45%	84.22%	84.14%
2 months	86.36%	84.04%	86.01%	85.95%
3 months	87.97%	85.52%	87.74%	87.70%
Expiration		Out-of-the-money		
1 month	56.94%	56.01%	56.37%	56.26%
2 months	67.02%	65.20%	66.59%	66.50%
3 months	70.94%	69.79%	70.63%	70.57%

Table 7 - Comparing values calculated by the models with the BM&FBovespa reference option premiums for the dollar put options considering the options' expiration dates

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
Expiration		In-the-money		
1 month	91.97%	91.34%	91.63%	91.58%
2 months	86.58%	85.82%	86.31%	86.26%
3 months	89.28%	88.12%	89.03%	88.99%
Expiration		At-the-money		
1 month	88.53%	85.42%	87.44%	87.31%
2 months	85.76%	82.78%	85.17%	85.07%
3 months	88.57%	85.49%	88.17%	88.09%
Expiration		Out-of-the-money		
1 month	81.97%	-13.16%	82.27%	82.23%
2 months	85.21%	65.58%	85.03%	84.97%
3 months	85.30%	75.43%	85.16%	85.11%

When studying the outlined scenarios, in the call options, the difference between the premiums calculated by the models and the reference option premiums decreases the longer the term to the expiration date is. In other words, the further off the expiration, the greater the comparison ratio between the models and the Reference option premiums. In the case of put options, this trend does not appear as clearly.

Another consideration regarding the findings relates to the number of instances where the models generated higher premiums than those of the BM&FBovespa reference option premiums. Tables 8 and 9 present the number of cases where this occurs.

Table 8 – Number of cases where the results generated by the models testing for call options were greater than the reference option premiums calculated by BM&FBovespa

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	71	53	71	70
At-the-money	56	50	54	54
Out-of-the-money	32	23	31	30

Total	159	126	156	154
% of the total	19.31%	15.30%	18.95%	18.71%

Table 9 - Number of cases where the results generated by the models testing for put options were greater than the reference option premiums calculated by BM&FBovespa

	Garman-Kohlhagen	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In the money	62	60	61	60
At-the-money	82	66	80	78
Out-of-the-money	81	80	81	81
Total	225	206	222	219
% of the total	28.77%	26.34%	28.38%	28.00%

The results reaffirm that BM&FBovespa calculates an reference option premium that is higher than the one calculated by the models, such that there is a greater prevalence of higher pricing in the out-of-the-money call options and in-the-money put options.

4.2. Options on Ibovespa futures

The findings for options on Ibovespa futures are presented in Tables 10 and 11.

Table 10 – Comparing values calculated by the models with the BM&FBovespa reference option premiums for call options on Ibovespa futures

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	99.12%	97.41%	111.95%	111.95%
Std. deviation	18.28%	19.43%	22.19%	22.19%
At-the-money	97.82%	95.53%	112.58%	112.58%
Std. deviation	30.20%	30.88%	34.42%	34.42%
Out-of-the-money	105.81%	105.07%	125.39%	125.37%
Std. deviation	52.36%	55.77%	60.98%	60.97%
Total average	101.26%	99.72%	117.53%	117.52%
Std. deviation	38.99%	41.19%	45.52%	45.51%

Table 11 - Comparing values calculated by the models with the BM&FBovespa reference option premiums for put options on Ibovespa futures

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	108.16%	107.91%	94.51%	94.51%
Std. deviation	22.89%	23.39%	23.60%	23.60%
At-the-money	97.70%	95.64%	82.71%	82.70%
Std. deviation	36.69%	37.85%	34.00%	34.00%
Out-of-the-Money	96.54%	92.27%	80.02%	80.00%
Std. Deviation	49.84%	50.13%	44.24%	44.24%
Total average	100.46%	98.23%	85.38%	85.37%
Std. deviation	38.43%	39.27%	35.49%	35.49%

A pattern in relation to the models is noticeable: whereas the Black Modified and Corrado-Su modified models converge with the reference option premiums, the jump-diffusion model points to overvaluation of call options and undervaluation of put options.

Additionally, it is notable that the models point to an overvaluation of the reference option premiums as the option is displaced toward being out-of-the-money, which is the same unidentified movement in the call options. The upward trend is observed in the standard deviation, as the option is displaced further toward being out-of-the-money is another point to be emphasized, since it points out a challenge or imbalance in the pricing of these options.

Unobserved, such as for dollar options, is a general convergence among the models, i.e., the average premiums calculated are not closely related, which makes it difficult to draw overarching conclusions about the findings.

Another way of analyzing the findings is by computing the values, while bearing in mind the expiration of each option, which thus enables picturing a possible effect of expiration on the difference between the reference option premium and the premium generated by the models. The results are exhibited in Tables 12 and 13.

Table 12 – Comparing values calculated by the models with the BM&FBovespa reference option premiums for call options on Ibovespa futures considering the expiration dates of the options

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
Expiration		In-the-Money		
1 month	98.03%	95.09%	106.17%	106.16%
2 months	101.79%	101.97%	117.85%	117.84%
3 months	97.97%	96.63%	118.02%	118.01%
Expiration		At-the-Money		
1 month	97.66%	94.53%	107.20%	107.21%
2 months	99.30%	99.82%	116.06%	116.05%
3 months	96.10%	91.51%	117.36%	117.34%
Expiration		Out-of-the-Money		
1 month	112.88%	111.47%	127.87%	127.87%
2 months	103.24%	104.58%	124.08%	124.06%
3 months	98.48%	96.11%	123.32%	123.31%

Table 13 - Comparing values calculated by the models with the BM&FBovespa reference option premiums for put options on Ibovespa futures considering the expiration dates of the options

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
Expiration		In-the-Money		
1 month	110.88%	109.79%	101.23%	101.23%
2 months	110.41%	112.11%	95.01%	95.00%
3 months	101.68%	100.54%	84.05%	84.04%
Expiration		At-the-Money		
1 month	97.12%	93.46%	85.98%	85.98%
2 months	98.93%	99.55%	81.85%	81.84%
3 months	97.26%	94.74%	78.21%	78.19%
Expiration		Out-of-the-Money		
1 month	97.28%	91.51%	83.73%	83.71%
2 months	95.83%	96.54%	77.90%	77.88%
3 months	95.93%	87.70%	74.83%	74.81%

Analyzing the outlined scenarios reveals that the ratios between the premiums calculated by the proposed models and the reference ones tend to be greater in contracts with a one-month term and decline as the term increases. In the call options, however, there seems to be greater stability among premiums, regardless of expiration.

Another aspect worth considering about the findings regards the number of instances where the models generated higher results than those of the BM&FBovespa reference option premium. The incidence of these cases is provided in Tables 14 and 15.

Table 14 – Number of cases where the results for call options generated by the models were higher than the prices listed in the BM&FBovespa reference option premiums

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	33	25	44	44
At-the-money	51	46	70	70
Out-of-the-money	54	53	71	71
Total	138	124	185	185
% of the total	45.24%	40.65%	60.65%	60.65%

Table 15 - Number of cases where the results for put options generated by the models were higher than the prices listed in the BM&FBovespa reference option premiums

	Black Modified	Corrado-Su modified	Merton's jump-diffusion model 1	Merton's jump-diffusion model 2
In-the-money	43	43	22	22
At-the-money	45	43	24	24
Out-of-the-money	38	28	20	20
Total	126	114	66	66
% of the total	50.19%	45.41%	26.29%	26.29%

The results point to a convergence between the BM&FBovespa reference option premiums and the premiums calculated by the Black Modified and Corrado-Su modified models. On the other hand, the findings highlight the inconsistency between the premiums of the jump-diffusion model and the reference option premiums.

4.3. Sample Comparison

It is necessary to compare the premiums from the models with the reference option premiums to verify whether the samples were statistically different. As such, a statistical test that provides robustness to the differences made explicit was chosen and carried out.

Due to the size of the sample, application of the t-test for difference in averages is suitable according to the small sample theory (SPIEGEL, 1971). This ensures that, even when using small samples, the statistic treatment is valid.

However, a parametric test, as in the t-test, assumes the normality of the samples, thus requiring analysis of the distribution. Testing for adherence to normality was carried out by referring to the tests by Shapiro and Wilk (1965) and Jarque and Bera (1987).

The Jarque-Bera statistic is based on differences between skewness and kurtosis coefficients of the distribution observed in the series and of the theoretical normal distribution. It plays a role in testing the null hypothesis that a sample was extracted from a normal distribution. If the statistic value is small, it signals rejection of the normality hypothesis. Here, as in Shapiro-Wilk's test, the null hypothesis is that the distribution is normal.

The results of both normality tests point to the non-normality of the distribution of calculated premiums for all models, whether call and put options for dollars and Ibovespa futures, and for the reference option premiums.

According to the results, a non-parametric comparison test between the samples was carried out, thus granting the findings greater robustness. Thus, each of the models was compared with the reference option premiums using the Wilcoxon Signed Rank test.

The results of the non-parametric test for sample comparison point to a statistical difference between the premiums found by the models and the reference option premiums for dollar call and put options. This shows an overvaluation of the reference option premiums published by BM&FBovespa when compared to the premiums calculated by the models.

Regarding call options for Ibovespa futures, there is no convergence in the results, given the fact that the Black modified and Corrado-Su modified models, despite providing premiums that are statistically different from the reference option premiums, do not demonstrate a clear pattern of over- or undervaluation. On the other hand, the jump-diffusion model points to an undervaluation of reference option premiums in relation to what has been calculated by the model. Thus, this makes it difficult to draw a wide-ranging conclusion about the results.

Similar to call options, the put options for Ibovespa futures do not show convergence among the models. Whereas the Black modified and Corrado-Su modified models show 1% alpha agreement with the reference option premiums, the jump-diffusion model signals their overvaluation. The results could not be generalized since the models do no converge.

5. CONCLUSIONS

Given market agents' use of the reference option premiums published by BM&FBovespa, it is necessary to use robust and reliable models for pricing options, thus allowing the investor to assess whether the negotiated price reflects the actual value of the option, or is over- or undervalued.

This paper used the Garman-Kohlhagen, Corrado-Su modified, Merton's jump-diffusion and Black modified models to draw a comparison between the BM&FBovespa reference option premiums for options on dollars and Ibovespa futures. Study scenarios were defined to this end, and their results compared with the reference option premiums calculated by the exchange from January 2006 to November 2014.

The findings enable us to conclude that the reference option premiums calculated by BM&FBovespa for dollar options are overvalued in the contracts studied, no matter if the options are in-, at-, or out-of-the-money. Moreover, it is worth noting that the models studied provide, in general, results similar among themselves, which corroborates the idea that the literature contains models that are able to replace the methodology applied by the exchange.

Specifically, regarding dollar options, just above 80% of the reference option premiums prices for call options were overvalued. For put options, the amount drops to just above 70%.

Concerning the reference option premiums for options on Ibovespa futures, the findings did not point in the same direction, since it is only possible to mention the undervaluing of call option premiums and overvaluing of put option premiums if they are only viewed through Merton's jump model. The other models do not show consistent conclusions.

Therefore, the results might suggest that BM&FBovespa could be including factors not addressed by the currency options pricing models in its calculations for the reference option premiums—an example could be the premium for the low liquidity of this type of derivative—or even a methodology unsuitable for the features of the series of underlying subject assets. Consequently, this study benefits investors and researchers by warning them of the inconsistencies in reference option premiums calculated by the stock exchange and the values obtained from the main models available to fund managers and researchers. This inconsistency has an impact on investment funds and other investors who use the reference option premiums in calculating their performance, since its estimation is inaccurate. Bearing this in mind, future studies are urged to seek factors that might explain the inconsistency.

Finally, the reference option premium is relevant to investment funds, which use it in the practice of mark-to-market. Moreover, premiums are used as indexes in balance sheets, since the value of a financial position that a company may henceforth have in the market is quantified based on the published reference option premiums. Incorrect pricing of premiums triggers unreal gains or losses for investment fund shareholders and encumbers the monitoring financial health of a company that uses financial derivatives based on an reference option premium.

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