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OVERVIEW OF RESEARCH ON STATISTICAL EDUCATION IN THE INITIAL AND CONTINUING EDUCATION OF MATHEMATICS TEACHERS

PANORAMA DE LA INVESTIGACIÓN SOBRE EDDUCACIÓN ESTATDÍSTICA EM LA FORMACIÓN INICIAL Y CONTINUA DE PROFESOES DE MATEMÁTICAS

PANORAMA DE PESQUISAS SOBRE EDUCAÇÃO ESTATÍSTICA NA FORMAÇÃO INICIAL E CONTINUADA DE PROFESSORES DE MATEMÁTICA

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Abstract
This article refers to a survey carried out in one of the stages developed in a Master's thesis work, in order to map the researches that have discussed Statistical Education in the Initial and Continued Training of Mathematics teachers, with a look at the Didactic-Mathematical Knowledge (CDM) and the Pedagogical Residency Program. The search was carried out on the following platforms: Catalog of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (CAPES), the Brazilian Digital Library of Theses and Dissertations (BDTD) and the *Site* Grupo de Investigación sobre Educación Estadística, and it was found that there were 14 works between theses and dissertations. Through this survey it was possible to identify the main questions raised about the theme, the problems pointed out by the researchers, and, mainly, the points raised by the participants regarding Statistical Education in teacher training. The results reveal that what the curricula propose for the training of Mathematics teachers, referring to statistical training, is not always achieved, which has generated insecurity in these teachers when working with this content in the classroom at the beginning of their career.

Keywords: Education; Statistics; Mathematics; Teachers; Knowledge.

Resumen
Este artículo se refiere a una encuesta realizada en una de las etapas desarrolladas en un trabajo de tesis de maestría, con el fin de mapear las investigaciones que han discutido la Educación Estadística en la Formación Inicial y Continua de profesores de Matemáticas, con

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una mirada al Conocimiento Didáctico-Matemático (CDM) y al Programa de Residencia Pedagógica. La búsqueda se realizó en las siguientes plataformas: Catálogo de Tesis y Disertaciones de la Coordinación para el Perfeccionamiento del Personal de Enseñanza Superior (CAPES), Biblioteca Digital Brasileña de Tesis y Disertaciones (BDTD) y el *Sitio* Grupo de Investigación sobre Educación Estadística, y se encontró que había 14 trabajos entre tesis y disertaciones. A través de esta encuesta fue posible identificar las principales preguntas planteadas sobre el tema, los problemas señalados por los investigadores y, principalmente, los puntos planteados por los participantes con respecto a la Educación Estadística en la formación docente. Los resultados revelan que no siempre se logra lo que los currículos proponen para la formación de profesores de Matemáticas, referidos a la formación estadística, lo que ha generado inseguridad en estos profesores al trabajar con este contenido en el aula al inicio de su carrera.

Palabras clave: Educación; Estadística; Matemáticas; Profesorado; Conocimiento.

Resumo
Este artigo se refere a um levantamento realizado em uma das etapas desenvolvida em um trabalho de dissertação de mestrado, no intuito de mapear as pesquisas que têm discutido a Educação Estatística na Formação Inicial e Continuada de professores de Matemática, com um olhar voltado para o Conhecimento Didático-Matemático (CDM) e para o Programa de Residência Pedagógica. A busca foi realizada nas plataformas: Catálogo de Teses e Dissertações da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Biblioteca Digital Brasileira de Teses e Dissertações (BDTD) e no *Site* Grupo de Investigación sobre Educación Estadística, e constatou-se a existência de 14 trabalhos entre teses e dissertações. Por meio deste levantamento foi possível identificar as principais questões levantadas acerca da temática, os problemas pontuados pelos pesquisadores e, principalmente, os pontos levantados pelos participantes referentes à Educação Estatística na formação docente. Os resultados revelam que nem sempre o que os currículos propõem para a formação dos professores de Matemática, referente à formação estatística, são alcançados, o que tem gerado insegurança nesses docentes ao trabalhar com esse conteúdo na sala de aula no início da carreira.

Palavras-chave: Educação; Estatística; Matemática; Docentes; Conhecimento.

Introduction

We often find tables, graphics or rates, as well as financial, economic and inflation indices, when we open the newspaper and magazine or when we watch the news or read some news in different communication media. Those are the ways to express certain information. The act of reading, interpreting and making decisions about such information demands citizens to have previous knowledge in order to do so.

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Statistics addition to Brazilian school discipline matrices, as well as its teaching, are not yet acknowledged. This subject is often found in the last pages of didactic books, and it is almost never addressed in teachers' teaching plans at Basic Education level. Whenever it is approached, it is always linked to some sort of mechanical, technical and instrumental treatment (SILVA, 2014).

Lopes and Carvalho (2009) highlight that, up to the 1950s and 60s, Statistics teaching was supported by deep concern with tools and methods necessary to solve problems found in different contexts. Teachers used to see Statistics as important instrument to allow several sectors of society to measure, describe and classify data and information. These authors also point out that

> Statistics relevance was limited to services provided to other knowledge fields. Consequently, at that time, its teaching would tend to reflections about this instrumental viewpoint, from which, Statistics is a set of strict mathematical notions and techniques that can use objective forms; the statistics activity is limited to these notions and techniques' formal and mechanical use (LOPES; CARVALHO, p. 78, 2009).

In the following decade, in the 1960s and 70s, statistics' aim was its mathematical aspects. At that time, there was great concern with detaching Statistics from Social Sciences, mainly when it comes to Statistics as discipline. It happened because of its objectivity and rigor features, and of its link to Mathematics, as shown by Lopes and Carvalho (2009). Yet, back in the old days, Statistics focused on solving similar problems, so students used to learn to recognize several problem types, to reproduce procedures and to apply concepts by valuing numerical aspects deriving from statistical tools.

From the 1970s and 80s onwards, we could witness the emergence of data exploratory analysis in Statistics teaching and learning. The computer society gave Statistics an outstanding position, since it is now seen as essential for citizens' scientific and ethical education. In the 1990s, Statistics was finally officially added to National Basic Education Discipline Matrix Parameters, and it demanded its emphasis since child education.

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Batanero (2013) warns that, although statistics is found in discipline matrices at all schooling levels, besides being an essential tool for individuals' personal and professional lives, some research highlight that many students, even at higher education, have wrong ideas about or a hard time properly interpreting statistical outcomes. According to this author, a likely explanation for such a scenario lies on routine teaching, which emphasizes formulas and definitions, without caring about activities demanding contextualized data and interpretation. Batanero points out that it is absurd teaching a meaningless Statistics, without considering its very nature.

Statistics Education emerged as response to teaching and learning concerns with Statistics. This new era started in the 1970s due to the need of finding ways to stop difficulties faced by higher education professors to teach concepts and procedures to Statistics users, as shown by Cazorla, Kataoka and Silva (2010). Therefore, Statistics Education is an Education field focused on investigating problems related to teaching and learning processes linked to Statistics, Probability and Combinatorics' concepts. This field addresses the cognitive and affective aspects of the teaching/learning association and the epistemology of statistics, probabilistic and combinatorics' concepts, as well as the development of teaching methods and materials aimed at developing Statistics Literacy.

According to Samá (2020), statistically literate individuals understand statistical concepts and can make critical analyses and arguments about data quality and reliability.

We understand the need of qualifying teachers to teach specific Mathematics and Statistics contents, so they can help students developing their statistical competences and literacy. We see the Pedagogical Internship Program, also known as PRP, as public policy conceived by the Coordination for the Improvement of Higher Education Personnel (CAPES). This policy focuses on Bachelor courses to provide scholarships to future teachers, professors and advisors focused on activities carried out in Basic Education schools, in partnership with Higher Education Institutions (IES).

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The main aim of the present study, which is a fragment of a Master's research called "Didactical-Mathematical Knowledge Mobilized by Advisors and Interns in the Pedagogical Internship Program: proposition for a work focused on Stat teaching". The goals were to seek research about Statistics Education in Mathematics Teachers' Initial and Continuous Education, and to analyze different approaches applied to this topic.

Methodology

Investigation processes demand listing productions that address, reason about and investigate the topic of interest. "Preliminary bibliographic screening will allow limiting the study site and [elaborating a] final problem definition" (Gil, p. 61. 2002).

According to Fiorentini, Passos and Lima (2016, p. 18), research mapping is

> the systematic process to screen and describe information about research on a specific study field, it has to cover a given space (location) and time lapse. This information regards this production's physical aspects (describing where, when and how many studies were produced within a given period-of-time, and who this production's authors and participants were) and its theoretical, methodological and thematic aspects.

We assessed three different environments to bibliographically screening Statistics Education in Mathematics teachers' initial and continuous education, namely:

- Catalog of Theses and Dissertations from the Coordination for the Improvement of Higher Education Personnel (CAPES)[3];
- Brazilian Digital Library of Theses and Dissertations (BDTD)[4];
- Research Group on Educational Statistics website[5].

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[3] Available at < https://catalogodeteses.capes.gov.br/catalogo-teses/#!/>. Accessed on March 18 and 19, 2023.
[4] Available at < https://bdtd.ibict.br/vufind/>. Accessed on March 20 and 21, 2023
[5] Available at < https://www.ugr.es/~batanero/pages/formacionprofesores.html>. Accessed on March 28 and 29, 2023

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These platforms were selected because they are relevant repositories: CAPES and BDTD are international catalogs and the website of the Research Group on Educational Statistics comprises international-level productions.

Descriptors "Statistics Education" AND "teachers' Education", "Didactical-Mathematical knowledge" and "Pedagogical Internship", were used to guide the search towards references in compliance with the herein proposed study.

The Catalog of Theses and Dissertations from the Coordination for the Improvement of Higher Education Personnel (CAPES) provided 40 studies related to descriptors "Statistics Education" AND "Teachers' Education". After their abstract were read, only 12 articles were selected to in-depth reading. Since the research subjects were Mathematics teachers, the remaining 28 studies were disregarded because they did not match our aims - they regarded other knowledge fields or populations.

Only 5 among the 12 research about Statistics Education in Mathematics Teachers' education approached educator professors' initial and continuous education. Therefore, 7 studies (4 dissertations and 3 theses) about this topic were selected.

A second search was carried out in the same databases to help better understanding productions about the herein proposed investigation, and they were based on the following descriptor: "Didactical-Mathematical knowledge"[6]. One must take into account that this Master's research focused on "identifying and understanding the Didactical-Mathematical Knowledge mobilized by PRP educator professors and interns for graphics and tables' teaching", on seeking studies about this topic that tend to help the development and awareness of aspects discussed within this context.

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[6] Didactical-Mathematical Knowledge (DMK) is a theoretical model whose analysis categories can be used as tools to identify and classify knowledge required for Mathematics' teaching. This model is based on the Ontosemiotic Approach (OSA). According to Godino (2009), OSA is a theoretical reference aimed at gathering different viewpoints and theoretical notions about mathematical knowledge, its teaching and learning.

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This search provided references from six theses that were previously read. However, only one research provided discussions linked to Didactical-Mathematical Knowledge (DMK) applied to Statistics Education, namely: the study by Carvalho (2017).

Descriptor "Pedagogical Internship" was used because the search was carried out in the Mathematics Sub-project of the Pedagogical Internship Program of Federal University of Minas Gerais. In total, 22 studies were found and their abstracts were read to find those actually regarding PRP, at CAPES scope. Only two of these studies regarded the Pedagogical Internship Program. However, none of them talked about Mathematics teachers' education, or about Statistics Education. Therefore, they were not selected for the present study.

The search process at the Brazilian Digital Theses and Dissertations (BDTD) database was based on descriptors "Statistics Education" AND "teachers' education", and it led to 37 results. In total, 12 of these studies approached Statistics Education in Mathematics teachers' education. Four (4) of these 12 studies were about Statistics Education in educator professors' Initial and Continuous Education. There were 4 more studies that had already been listed at CAPES' search. Thus, only 4 studies were included in the present literature review.

Descriptor "Didactical-Mathematical Knowledge" led to 5 results, but they had already been selected in previous searches. None of them was about Statistics Education. The search in the Pedagogical Internship Program (PRP) used descriptor "Pedagogical Internship" and it led to 54 studies. However, there were studies whose topics did not concern PRP, but medical internship programs, among others – they did not meet the present research aim.

If one takes into account that DMK is closely linked to the Ontosemiotic Knowledge Approach and to Mathematics' Teaching, and that PhD Professor and researcher Carmen Batanero is among those who have drafted the system known as Mathematics assignment analysis category, which is based on elements of the Ontosemiotic Approach applied to Mathematics Knowledge and Assignment (OSA), we see the relevance of the search carried out on the website of the Research Group

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on Statistical Education. It is so, because this group is dedicated to trigger discussions on Statistics Education. These discussions approach productions about different aspects forming the didactical procedures applicable to probability and statistics (students' concepts and reasoning, epistemological analysis, classroom materials and resources).

We created a favorable environment for those dedicated to assess this topic. The search on the repository website meant a significant return to studies investigating and discussing Statistics Education. In total, 11 of the 55 studies approached Mathematics teachers' education. Nine (9) of these studies regarded Primary School teachers' education (6- to 11-year-old students) and 2 studies approached Secondary School teachers (12- to 16-year-old students).

There were 55 theses and dissertations available when the search on GEEUG website was carried out, but after checking on their abstracts, only 2 of them were selected: Contreras (2011) and Gea (2014). Both studies addressed Mathematics teachers' knowledge.

The search allowed finding a theoretical *construct* based on 14 studies about Statistics Education in Mathematics teachers' education. Some of them regarded initial education and the others focused on these teachers' continuous education They provided considerations about discipline matrices, and the perceptions by students, Basic Education teachers, professors and coordinators.

Some studies were similar to each other, but they were selected and will be introduced below - they were split into categories.

Chart 1: Sets of research analyzed based on category

Category	Research
Perception by students, professors and coordinators in Statistics Education Schools.	Costa (2007), Fuchs (2013), Ribeiro (2004).
Statistics Education and Discipline matrix	Silva (2014)
Statistics Education and education practices of Mathematics professor and students	Damin (2018), Souza (2019)
Statistics literacy	Goulart (2015), Rodrigues (2018), Fernandes (2020)
Statistics Education and knowledge of Mathematics professors	Malara (2008), Carvalho (2017), Gea (2014), Sera (2016), Contreras (2011)

Source: Elaborated by the authors (2023)

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Costa (2007), Fuchs (2013) and Ribeiro (2004) investigated the perceptions, concepts, attitudes and ideas by students, professors and coordinators about Statistics and Probability teaching in Basic Education and in teachers' education courses. Silva (2014) assessed Basic Education discipline matrices based on Higher Education discipline matrices. Damin (2018) and Souza (2019) were concerned with undergraduates and Bachelors' Statistics and Probability teaching and learning. Malara (2008), Carvalho (2017), Gea (2014), Sera (2016) and Contreras (2011), in their turn, sought to understand and feature the necessary knowledge for Mathematics teachers. Goulart (2015), Rodrigues (2018) and Fernandes (2020) aimed at developing statistics and probability literacy in teachers' education from the perspective of investigating a more specific mathematical/statistical knowledge set.

Overall, based on these studies, their subjects see Statistics Education as extremely necessary for citizens' formation, for them to understand and make decisions based on statistical information outspread in different media.

Teachers report to have a hard time understanding the Statistics concepts they are supposed to teach their students about. They state to be unprepared to such a task. They disclose that Statistics teaching is a challenge for many teachers (Artega, 2011). Therefore, we started from this very issue to analyze the contributions from the 14 selected studies about Statistics Education in mathematics teachers' initial and continuous education.

Students, professors and course coordinators' perception about Statistics Education

Costa (2007), Fuchs (2013) and Ribeiro (2004) investigated the perceptions, concepts, attitudes and ideas by students, professors and coordinators about Statistics and Probability teaching in Basic Education and in teachers' education courses.

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The study by Costa (2007) stands out among Brazilian research about stochastic education[7] in Mathematics teachers' education, since she investigated the perception by teachers working with Basic Education and by higher education professors. This author analyzed how school teachers reason about Statistics Education's addition to the discipline matrix.

She applied a questionnaire to basic school teachers who were working in public and private schools in Jundiaí region, São Paulo State, to identify the ones working with the stochastic approach, from her methodological perspective. In total, 30 teachers answered the questionnaire. The researcher realized that she needed to hear those in charge of educating this population after recurrent reports on flaws in teachers' initial education focused on teaching Statistics and Probability in Basic Education. She interviewed 3 professors from different institutions to hear them about Statistics' teaching in Mathematics teachers' initial education.

Their theoretical input aimed at discussions about the presence in and relevance of the Stochastic approach for Basic Education discipline matrices. They made comments on connections between Critical Mathematics Education and Statistics Education. Their reflections regarded professors' education, mainly educator professors. Fuchs (2013) introduced discussions about Statistics Education in Mathematics Teachers' initial education and about its teaching in Basic Education, because she was encouraged to analyze Statistics teaching ideas in the education of future Mathematics professors for public and private Bachelor's courses in Rio Grande do Sul State. Initially, she carried out the documental analysis of Pedagogical Course Project (PPC), course curriculum matrices, syllabi and teaching plans corresponding to Statistics and Probability subjects offered in the courses. It was done to complete the set of data production instruments. She applied questionnaires

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[7] The integrated work between Statistics and Probability has been called stochastic in several countries worldwide (COSTA; NACARATO, 2011). COSTA, A.; NACARATO, A. M. A Estocástica na Formação do Professor de Matemática: percepções de professores e de formadores. Bolema, Rio Claro (SP), v. 24, n. 39, p. 367-386, ago. 2011. Available at: file:///C:/Users/Flavia/Downloads/vbenites,+Artigos+2.pdf. Accessed on September 18, 2023.

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to coordinators and professors who teach Statistics and Probability disciplines in Mathematics Bachelor Schools, in 17 public and private universities in Rio Grande do Sul State.

Ribeiro (2004) assessed 74 undergraduates in Mathematics from a private institution in Espírito Santo State who accepted to answer the questionnaire with open and closed questions. This questionnaire aimed at featuring participants' personal data and data about their ideas on several aspects linked to the herein investigated topic. It allowed creating an attitudes-scale for Statistics to assess these students' attitudes and ideas towards this content.

Based on analyses and reflections in the aforementioned studies, direction boards of Mathematics Bachelor Schools are concerned with qualifying teachers with professional profile focused on Basic Education school's demands for citizenship formation, with emphasis on future teachers' critical and reflexive professional profile.

However, these authors' studies show that focus on both stochastic disciplines and on the teaching performance of educator professors have gaps when it comes to students' qualification to work with Statistics Education in school environment. Aspects inherent to education processes based on statistical and probabilistic knowledge are disregarded, and it can lead to basic Education Mathematics teachers' hard time approaching these concepts.

According to Fuchs (2013), there is still a lot to be discussed and investigated, so that both initial education courses can get excellence in Statistics Education, mainly in developing competences to qualify statistically literate professionals.

Statistics Literacy

The sense of literacy has been changing among researchers, overtime, due to updates in Statistics Education. Over time, the view of literacy has undergone many variations among researchers, due to the advancement of Statistical Education.

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According to Wodewotzki and Jacobini (2011), literacy is the ability to read, understand, analyze and assess written texts. Statistics literacy, in its turn, refers to the capacity of arguing by properly using statistics technologies. These skills include the ability to organize data, plot and present tables, and to work with different data representations (CAMPOS; WODEWOTZKI; JACOBINI, p. 23, 2011).

Based on Gal (2004), statistics literacy refers to 2 interrelated components:

> 1. A person's ability to critically interpret and assess statistical information, arguments related to research data and stochastic phenomena often found in several contexts;
> 2. A person's skill to discuss or inform its reactions to this statistical information, such as its interpretations, opinions and learning about this information's meaning.
> (GAL, 2004, Apud. CAMPOS; WODEWOTZKI; JACOBINI, p. 27, 2011).

Goulart (2015), Rodrigues (2018) and Fernandes (2020) conducted their research focusing on statistics and probabilistic literacy in teachers' education to help them developing this skill.

Goulart (2015) has set associations assumingly found between Statistics teaching in Basic Education and Statistics teaching in Mathematics Bachelor Schools. His goal was to boost teachers' education for Statistics Literacy. He also made a documental analysis of the following documents: National Curricular Parameters (PCN) of the third and fourth cycles of Elementary Education (Mathematics), PCN of Secondary Education (Natural Sciences, Mathematics and their Technologies), PCN+ (Natural Sciences, Mathematics and their Technologies), the Curricular Guidelines for Secondary Education (Natural Sciences, Mathematics and their Technologies), the National Curricular Guidelines for Mathematics, Bachelor's and Degree Courses (DCN-BL) and the National Curricular Guidelines for the Training of Basic Education Teachers, at Higher Level, Course degree, full degree (DCN-FP), as well as the following large-scale exams: *SAEB, Prova Brasil, ENEM and ENADE,* and two textbook collections approved by PNLD - intended for Basic Education.

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Goulart (2015) assessed aspects assumingly found in these documents that could contribute to Statistics Literacy skills, and to statistical thinking and reasoning, based on the anthropological Didactics Theory (ADT) as theoretical reference.

Fernandes (2020), in his turn, assessed whether the participation of teachers working in the Final Years of Elementary School in continuous qualification courses, in addition to de Curcio's (1989) Graphic Understanding and to Gal's (2002) Statistics Literacy, could help developing a more effective Statistics Education. Fernandes (2020) elaborated and implemented a professional enhancement course focused on Statistics Education, which comprised Statistics and Probability assignments, scientific-articles' reading and statistical investigations. He counted with 12 teachers from Paraná State Public Teaching Network. His qualification course encompassed 15 classes (4-h long each) and its final assignment was to elaborate a teaching proposal.

Fernandes' (2020) wrote about Mathematics teachers' education, national discipline matrix guidelines for teachers' education, teachers' knowledge, Statistics Education, graphics' understanding and statistical skills, mainly Statistics Literacy.

Rodrigues (2018) aimed at analyzing probability and randomness concepts of teachers working in Basic Education when they face questions about this topic. He used categorization probabilistic concepts by Azcárate (1995), Cardeñoso (1998) and the definition of Probability Literacy proposed by Gal (2005).

Rodrigues (2018) made the documental analysis of National Curricular Parameters (PCN) and the National Common Curricular Base (BNCC) to find likely similarities and updates in the transition from the 1st to the 2nd document, when it comes to approach probability topics in Basic Education. The "probability Concept Questionnaire" was applied to 41 Elementary School Mathematics Teacher (final years of it) in order to identify this population's concepts of probability. The adopted methodology was based on multi-dimension data analysis, which was substantiated by implicative and cohesive analysis applied to teachers' answers. Hierarchical, Implicative and Cohesive Classification (CHIC) software was used to plot cohesion and implication graphics associated to the assessed variables.

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According to these authors, the analyzed documents do not boost Statistics Literacy development, or have any associations between Statistics teaching in Basic Education and Statistics Teaching in Mathematics Bachelor Schools focused on enhancing teachers' education for Statistics Literacy.

Statistics Education and Discipline Matrix

Silva (2014) investigated Statistics and Probability teaching education in Mathematics Bachelor Schools in Brazil by analyzing discipline matrices, Pedagogical Political Projects, Discipline application programs and questionnaire application to courses' coordinators. His study was substantiated by discipline-matrix discussions by Sacristán (2000), basic knowledge for teaching by Shulman (1986, 1987), and by Gal and Garfield (1997) studies to assess the targets to teach these components.

Silva (2014) focused on the distance between Statistics teaching in Basic Education's discipline matrices and those of Mathematics' Bachelor Schools.

Silva (2014) aimed at initially mapping 78 courses distributed into 48 Higher Education Institutions (IES) in Brazil. However, due to hard time getting answers, she had to analyze the discipline matrices of more than 200 courses. This author analyzed discipline matrices, Political Pedagogical Projects, and disciplines' programs to analyze Statistics and Probability teaching. After making such a mapping and analyzing these documents, Silva (2014) applied a questionnaire to the assessed courses' coordinators.

Her results have shown that discipline matrix components for concepts' formation in Statistics and Probability are mandatory in matrices of Mathematics Bachelor Schools, regardless of region, municipality, state or teaching network. All the analyzed courses focus on matrices based on these contents' conceptual teaching.

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However, Silva (2014) points out that, despite the presence of Statistics and Probability as content, the structure of some courses still advocate that teachers' education must target conceptual knowledge. There were evidences of curricular components related to discipline matrix, epistemology, history and philosophy, research or research methodology and psychology, but not differentiated aspects capable of influencing teachers' education for Statistics teaching. Silva (2014) also highlights that, by analyzing these documents, she faced the prescriptive discipline matrix, and it allows saying that such prescriptions remain in nowadays matrices. This element was stressed in the studies by Goulart (2015), Rodrigues (2018) and Fernandes (2020), according to whom, not all contents in discipline matrices are actually followed in Mathematics teachers' initial education courses.

Statistics Education and Education practice by Mathematics teachers and undergraduates

Damin (2018) analyzed contributions from Statistics Teaching Projects to Mathematics teachers' education. He developed statistics skills and competences for teaching practices, based on Statistics and Probability assignments. It must be achieved by elaborating and implementing a Statistics teaching project composed of Statistics and Probability assignments, scientific-articles' reading and discussions, and statistical investigations. His project counted on 11 undergraduates from a public university in Paraná State - it comprised 35 classes. Participants were asked to elaborate a teaching assignment.

In order to reach his goal, Damin (2018) used theoretic bases on Mathematics teachers' education, national discipline matrix guidelines for teachers' education, teachers' knowledge, Statistics Education field and statistical skills.

Souza (2019) identified how teachers understand information available in graphics, and their concepts of Statistics Literacy and Rural Education, by discussing the relevance of working with Statistics Education, with graphics' interpretation from

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the Statistics Literacy perspective, with analysis of pedagogical assignments in Rural Education, with qualification workshops carried out with participant teachers. She carried out semi-structured interviews with five Mathematics teachers who work in the final years of Rural Elementary Schools. Three (3) meetings were scheduled and they were conducted in workshop format. They allowed teachers to engage in discussions and to have the opportunity to develop Statistics Literacy.

Mathematics teachers' Statistics Education and knowledge

CAPES, BDTD and GEEUG website led to five studies focused on investigating Mathematics teachers' knowledge applicable to Statistics and Probability teaching: Malara (2008), Carvalho (2017), Gea (2014), Contreras (2011) and Sera (2018).

Chart 2: Research aims and Country of origin.

Research	Aim	Country of origin
Malara (2008)	Observing, understanding and featuring educator professors' knowledge seen as necessary for the pedagogical practice of teachers in charge of Statistics Introduction Courses	Brazil
Contreras (2011)	Providing information about mathematical knowledge of future primary and secondary education teachers about conditional probability	Spain
Gea (2014)	Exploratory study to assess didactical-mathematical knowledge about correlation and regression.	Spain
Sera (2016)	Discussing the teaching/learning process applied to reading and plotting graphics available for Basic Education Mathematics teachers	Brazil
Carvalho (2017)	Investigating how a qualification program helps building didactical-mathematical knowledge about probability among mathematics teachers working in the final years of elementary school	Brazil

Source: Elaborated by the author (2023)

Studies by Contreras (2011), Gea (2014) and Carvalho (2017) address Didactical-Mathematical Knowledge linked to theoretical references of the Didactical-Mathematical Knowledge Model (DMK), by Juan Godino and collaborators, which are the same references substantiating our own work.

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Sera (2016) does not approach DMK, but introduces mathematical knowledge based on categories set by Shulman and Ball, and Thames and Phelps (2008), who have followed the DMK approach. This author discusses the reflexive teacher profile highlighted by Zeichner (1993).

Sera (2016) investigated ways to broaden knowledge bases of Basic Education Mathematics Teachers based on continuous formation, with emphasis on teaching focused on statistics graphics' reading and construction for students in the final years of elementary school and in high school. Its aim is to share reasoning about Statistics Literacy and difficulties in teaching concepts related to this topic. His research was carried out with teachers working in the final years of Elementary School and High School. The study was split into two stages. The first stage was a diagnostic study based on applying two questionnaires. The first questionnaire aimed at gathering information about participants' profile and at acquiring previous knowledge about statistical graphics' plotting and interpreting. The second questionnaire focused on analyzing pedagogical knowledge about teachers' teaching practices on this topic.

The second phase regarded a series of assignments for participant teachers. Their aim was to encourage discussions and debates about the teaching/learning process applied to statistical graphics' plotting and interpreting.

Carvalho (2017) also carried out a research on continuous education by investigating how qualification programs help producing didactical-mathematical Knowledge on probability for teachers working in the final years of Elementary School. Carvalho (2017) implemented a course named 'formation course' for 40 teachers. It took place through 7 classes and comprised adjustments in the sequence of assignments suggested by Bryant and Nunes' teaching program on Probability and Risk, and on assignments in the literature about reflections on both probability and its teaching.

Malara (2008) sought to better understand and feature knowledge educator professors see as essential for the pedagogical practice of teachers working in Statistics' introduction courses. This inquiry targeted processes to learn this discipline, with emphasis on statistical thinking. This author addressed Mathematics

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teachers' knowledge by approaching pedagogical beliefs, concepts and practices that are substantiated by other theoretical inputs, such as Ponte, Thompson, Tardif, among others.

Malara (2008) also searched the literature about the knowledge required for teachers' professional qualification and development. She aimed at broadening results about the knowledge of teachers working with Statistics' introduction courses. Subsequently, she held semi-structured interviews with four educator professors.

Gea (2014) carried out an exploratory study to assess the Didactical-Mathematical Knowledge of future Secondary Education and Bachillerato[8] teachers about the correlation and regression available in didactic books. This author initially analyzed 16 Bachillerato didactic books (8 on Sciences and Technology and 8 on Humanities and Social Sciences) to go deep in different mathematical-object types seen as research theoretical references. She also assessed how they are introduced and interrelated. The second and third stages, which were called studies, aimed at planning didactical assignments to be accomplished in the practical sessions. They combined the assignment to a statistical analysis project, to collective discussions about project solutions (Study 2) and to didactical adjustment analysis (Study 3).

Contreras (2011) provided information on the mathematical knowledge of future Primary and Secondary Education teachers about Conditional Probability teaching.

Contreras (2011) also split his work into stages/studies. He assessed the knowledge of future Secondary Education and Bachillerato teachers in studies 1 and 2 to collect information about the need of qualifying teachers working at any schooling levels, other than higher education – he assessed their mathematical and didactical knowledge. Studies 3 and 4 aimed at analyzing some didactical resources assumingly useful to fulfil these teachers-qualification needs in either Primary or

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[8] It regards an education stage attended abroad. School system in Spain is based on the following organization: Early Child Education (3 to 6 years old), Primary Education (6 to 11 years old), Mandatory Secondary Education-ESO (12 to 16 years old). Bachillerato is the last year of ESO and it aims at preparing students to the university.

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Secondary Education. Internet and Probability history resources allowed organizing mathematical and didactical situations and reflections for teachers. Study 5 introduced the outcomes of teachers' qualification workshops, based on one of these resources.

The aforementioned studies have shown that discussions and reasoning taking place during education time have broadened the very basis of teachers' knowledge on how to teach this topic. Results in the current study also point towards the need of having qualification processes to gather different approaches, strategies and materials for teaching/learning processes applicable to Statistics' concepts (Sera, 2016).

According to Carvalho (2017), initial knowledge by the group of participating teachers about probability and its teaching is not enough to a reliable teaching/learning process focused on Elementary School students.

Besides insufficient knowledge for a satisfactory teaching/learning process, Contreras (2011) also showed that 1 in each 3 interviewed students had a wrong concept of statistics teaching. He also showed the high rate of future teachers who will present fake information or bias related to Conditional Probability, mainly when it comes to the time axis, to basic rates, joint probability and conditional.

Gea (2014) highlights in-depth information about the meaning of institutional references in didactic books that provide criteria for their enhancement and use by teachers. Other contributions of it regard assignments designed to future teachers, as well as detailed analysis of their knowledge and examples of contexts when teachers put at practice different profiles of such a knowledge. These profiles can be useful for teachers' education courses.

This same author highlights that didactical adjustment criteria proposed by Godinho and collaborators open room for a quite useful analysis instrument adopted to develop different knowledge types. She also points out that participants value the use of technologies to approach Statistics in their work.

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Final considerations

The main aim of the present study was to investigate research about Statistics Education in the Initial and Continuous Education of Mathematics Teachers by assessing publications available in CAPES Catalog of Theses and Dissertations, BDTD and in the website of *"Grupo de Investigación sobre Educación Estadística (GEUUG)"*.

In order to reach our goal, we analyzed 14 studies found the herein assessed databases. This movement allowed highlighting important Statistics Education features for Mathematics teachers' education, as herein pointed out.

Aspects observed through the mapping process allowed highlighting that Brazilian research is mainly carried out in Southeastern, Southern and Northeastern Brazil (most of it is carried out in São Paulo State) and that international research was carried out in Granada, Spain. Most of these research result from Mathematics Education programs and from Post-Graduation Programs in Mathematical and Technological Education. These studies regard undergraduates in Mathematics Bachelor Schools and Mathematics teachers working in Basic Education.

Among the assessed research, 9 focused on Statistics Education at initial education stages and 5 aimed at Mathematics teachers' continuous education. Their analysis showed concern with citizens' statistical education at all schooling levels. This concern gets stronger when it comes to teachers' initial education, because such an issue focuses on qualifying professionals who will deal with people's education.

Researchers highlight the potential of discussions and reasoning to broaden teachers' knowledge to teach Statistics. Results in the current study also pointed out the need of having qualification processes substantiated by connections among different approaches, strategies and materials developed for teaching/learning processes applicable to Statistics concepts.

They also show that what discipline matrices suggest for Mathematics teachers' education about Statistics Education is not always accomplished. Having the ears open is essential for this research type, so one can better understand how

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these teachers' initial education echoes on the schools they work at and on how they work with Statistics and Probability contents.

The research with educator professors and Mathematics Bachelor Schools' coordinators also showed the need of thinking about the initial and continuous education field.

Analyses and reflections resulting from the present investigations have shown that Mathematics Bachelor Schools are concerned with qualifying teachers based on professional profiles linked to Basic Education school demands, when it comes to education for citizenship, with emphasis on teachers' critical and reflexive professional profile.

However, the aforementioned studies highlight the stress given to stochastic disciplines and to teaching practices develop by educator professors, which show gaps in Bachelor's education focused on working with Statistics Education in school environment. This finding led to the conclusion that aspects inherent to education processes linked to statistics and probability knowledge have been disregarded, and it can lead to Basic Education Mathematics teachers' hard time approaching these concepts.

The analysis of discipline matrices and documents also pointed out other elements. None of these ecosystems boosts Statistics Literacy development. According to Goulart (2015), there are no associations between Statistics Education in Basic Education and Statistics Teaching in Mathematics Bachelor courses focused on enhancing teachers' education for Statistics Literacy.

Even if the teacher does not have initial education good enough to make it comfortable to teach Statistics contents, Costa (2007) shows that teachers seek to add Stochastics to their classes - they are mainly supported by didactic and para-didactic books. Educator professors acknowledge the gaps in teachers' education, as well as conceptual and epistemological issues in didactic books, with emphasis on small flexibility in nowadays guidelines set for Bachelor courses. These professors also point out the need of reformulating these guidelines to meet demands of future teachers' statistical thinking, so they can feel ready to work with this content in Basic Education.

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With respect to knowledge necessary for Mathematics teachers, there were evidences about different knowledge types, namely: those resulting from their experience as students, from concepts about professors' function, from learning concepts, from how to teach procedures that define paradigms related to knowing a specific content, from mobilization for a given knowledge, from educator professors' attitude, from factors interfering with teachers' practices and from knowledge about students' difficulties.

According to Studies focused on PRP, discussions are not scarce, although they were not found in research focused on Statistics Education programs, or on Mathematics teachers' education, in the herein assessed databases. This scenario shines light on the need and urgency of investigating this little explored field, since it regards public policies aimed at teachers' initial education for their future teaching activities.

Accordingly, the herein presented study opens room for discussions and reflections. One of them lies on the need of further research aimed at Statistics' teaching in Mathematics teachers' initial and continuous education. This discipline's teaching must start at Child Education and it must aim at promoting Statistics Literacy and other Statistical skills, namely: Statistical Thinking and Reasoning.

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